6.11 Set Theory: Symmetric Sets

Sets can have transpositional symmetry, inversional symmetry, or *both* types of symmetry.

Transpositional Symmetry

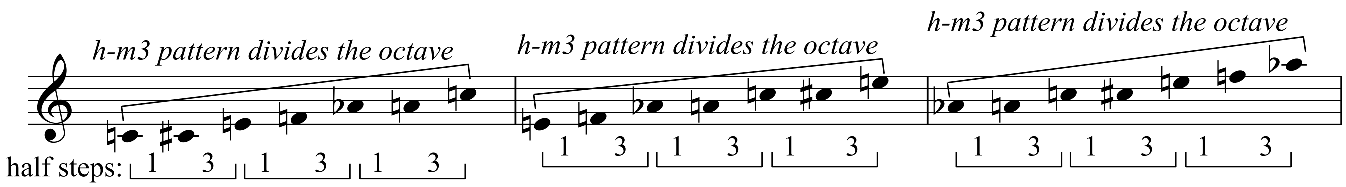
A set is ***transpositionally symmetric*** (or ***transpositionally symmetrical***) if its normal order divides the ***octave*** with a ***repeating pattern*** of half steps, like 1-5--1-5 or 1-3-2--1-3-2 or 4-4-4. The repeating pattern means you can start on different note(s) in the set and keep the pattern.

REMEMBER: Transpositionally symmetric sets have a repeating pattern of half steps and an unbreakable tie for normal order, and they reproduce themselves when transposed up or down by a particular interval(s).

Example of Transpositional Symmetry

The set below is ***transpositionally symmetric***. We know this because there is an ***unbreakable tie*** for the normal order. Comparing the second-to-last intervals, etc., doesn't help, because the interval structure is exactly the same for all three options.

Doubling the lowest notes allows us to see the repeating pattern of half steps that divides the octave.
All transpositionally symmetric sets divide the octave using some sort of repeating half step pattern.



Transpositionally symmetric sets are also called ***modes of limited transposition*** if they have more than 5 notes. They reproduce their pitch classes when you transpose them by a particular interval called the interval of transpositional symmetry. ***The interval of transpositional symmetry is always equal to the total size of the repeating pattern in half steps and is always less than an octave.***

The interval of transpositional symmetry for the example above is 4 half steps, or a major third, since the repeating pattern [1,3] spans 4 half steps. As shown above, transposing the leftmost normal order up a major third to E or down a major third to Ab gives the same pitch classes in a different order. This particular set is a ***hexatonic scale*** (h-m3-h-m3, etc.; see *6.4 More Contemporary Scales*).

Inversional Symmetry

A set is ***inversionally symmetric*** (or ***inversionally symmetrical***) if there is a tie for ***best normal order***. Inversionally symmetric sets have consecutive interval patterns that are ***palindromic*** (the same forwards and backwards) like 1-3-3-1 or 1-4-1, **OR** that divide the ***octave*** with a pattern of half steps that is ***palindromic***, like 1-5-5-1 or 3-6-3 or 4-4-4. This type of pattern means you can build the set upwards or downwards (and downwards means *inversion* here) and get the same interval pattern.

REMEMBER: Inversionally symmetric sets have a palindromic pattern of half steps and an unbreakable tie for BEST normal order, and the inversion will reproduce the original set when transposed up or down by a particular interval(s).