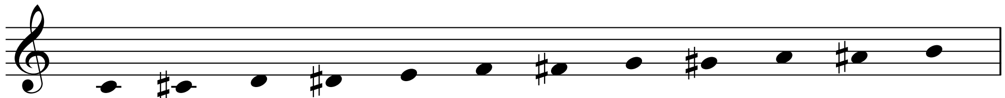
6.10 Set Theory Simplified

Pitch Classes

Pitch class = A *category* of notes including all possible octaves and enharmonic spellings of a particular pitch. There are only 12 pitch classes. For instance, C, B♯, and D𝄫 in *any octave* are all in one ***pitch class***. ***Pitch classes*** are normally numbered from 0 to 11, starting on C:



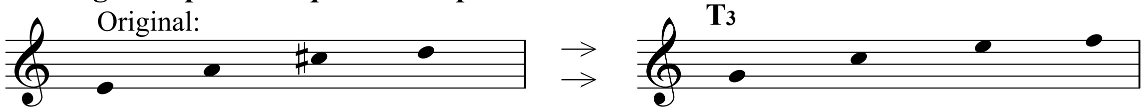
Pitch class numbers: 0 1 2 3 4 5 6 7 8 9 "T" =10 "E" =11

Transposition of Sets (T_n)

1. A ***set*** is any unordered group of unique (no-repeats) pitch classes. For example, D, F, A♯ is (2, 5, T).
2. To transpose a set ***up*** by *n* half steps, ***add n*** to each pitch class in the set.
3. To transpose a set ***down*** by *n* half steps, ***subtract n*** from each pitch class in the set.
4. If you get a number larger than 11 or smaller than 0, ***add or subtract 12*** to get a valid pitch class number.

Example showing transposition *up* 3 half steps:

Original: → **T_3**



Pitch class: 4 9 1 2

New pitch class: 7 0 4 5

(Calculation:) (4+**3**=7) (9+**3**=12, 12-12=0) (1+**3**=4) (2+**3**=5)

Example showing transposition *down* 3 half steps:

Original: → **T_{-3}**

Pitch class: 4 9 1 2

New pitch class: 1 6 T E

(Calculation:) (4-**3**=1) (9-**3**=6) (1-**3**=-2, -2+12=10) (2-**3**=-1, -1+12=11)

Inversion of Pitch Classes: (12-x)

In set theory, ***inversion*** means the same ***number of half steps*** in the ***opposite direction***.
The ***inversion of a pitch class*** is the pitch class that is the same ***number*** of half steps away from C, but in the ***opposite direction***. If you get a number less than 0, ***add 12*** to get a valid pitch class number.

ORIGINAL: 4 half steps <u>above</u> C
0+**4** = Pitch class 4 = E

INVERSION: 4 half steps <u>below</u> C
0-**4** = -4, but **-4**+12=Pitch class 8=A♭

C=0 E=4 C=0 A♭=8

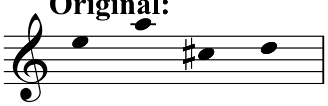
REMEMBER: ***For any pitch class x, inversion = (12-x)*** [since (0-x)+12 = (0-x+12) = (12-x)].

Inversion of Sets (TnI)

Inversion of ***sets*** (TnI) is a two-step process: ***first invert*** each pitch class (12-x), ***then transpose*** (Tn).

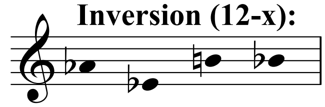
Example: T2I

Original:



Pitch class: 4 9 1 2

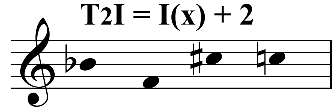
→ **Pitch-class Inversion (12-x):**



8 3 E T

→ *(add 2 to each to transpose...)*

T2I = I(x) + 2



T 5 1 0

<u>**OPTIONAL shortcut:**</u> Since I(x)=(12-x), TnI(x) = Tn(12-x) = (12-x)+n = (12-x+n) = (n-x+12).
SO **TnI(x) = (n-x)**, if you remember to add/subtract 12 as needed to keep pitch classes as 0 to 11.

6.10 continues on next page...

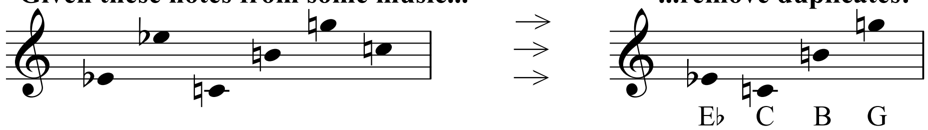
6.10 Set Theory Simplified

Normal Order Method 1: The Full Method

Normal Order = A standard format for listing pitch class sets; normal order is useful for comparing sets. The four steps for finding normal order are to ***remove duplicates***, ***list pitches low to high*** in one octave, ***select the smallest outer interval***, and ***break the tie*** if necessary.

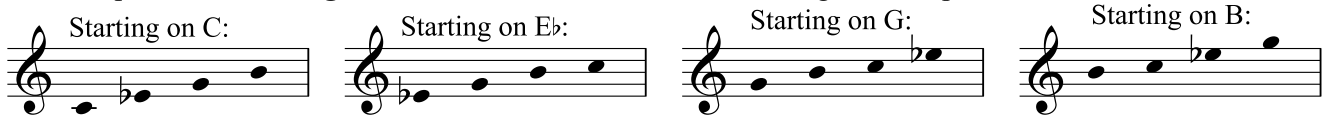
1. **Remove duplicated notes** to get a list of unique pitches.

Given these notes from some music... **...remove duplicates:**

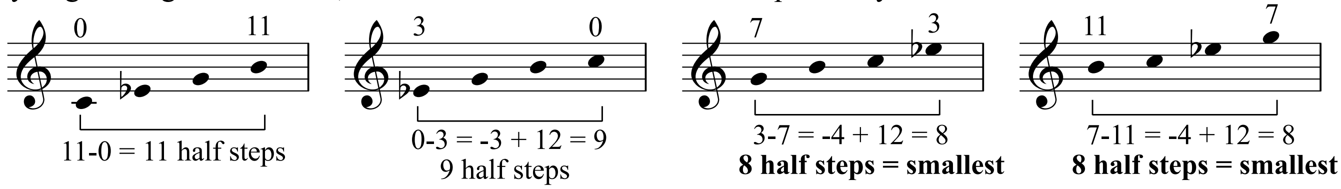


2. List the pitches **low to high in one octave**. Make versions starting on each pitch in the list.

Starting on C: Starting on E♭: Starting on G: Starting on B:



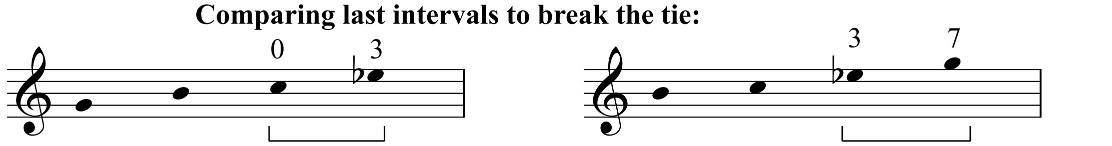
3. For each version, find the ***smallest outer interval*** by subtracting the bottom pitch class from the top. If you get a negative number, add 12 to correct it. Proceed to step #4 only if there is a tie.



Both have 8 half steps -- a tie! So, go to step 4.

4. If there is a ***tie*** for ***normal order*** (as in this example), select the version with the largest interval from the ***second-to-last*** to the ***last*** pitch class. If there is still a tie, check the ***third-to-last*** to the ***last***, and so on.

Comparing last intervals to break the tie:

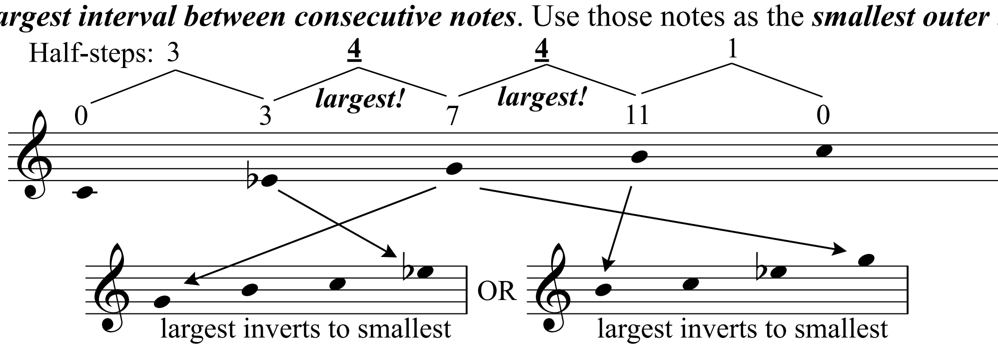


3-0 = 3 half steps

7-3 = 4 half steps = <u>larger</u> than C up to E♭,
so NORMAL ORDER=[B,C,E♭,G]

Normal Order Method 2: Shortcut with Pitch Class Numbers (RECOMMENDED!)

1. Remove duplicated notes and list the pitch class NUMBERS in order, ***smallest pitch class number first***.
2. Duplicate the lowest pitch class at the end. (Do NOT make versions starting on each pitch.)
3. Find the ***largest interval between consecutive notes***. Use those notes as the ***smallest outer interval***.



4. Use step 4 from the "full" method above to break any ties.

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6.10 Set Theory Simplified

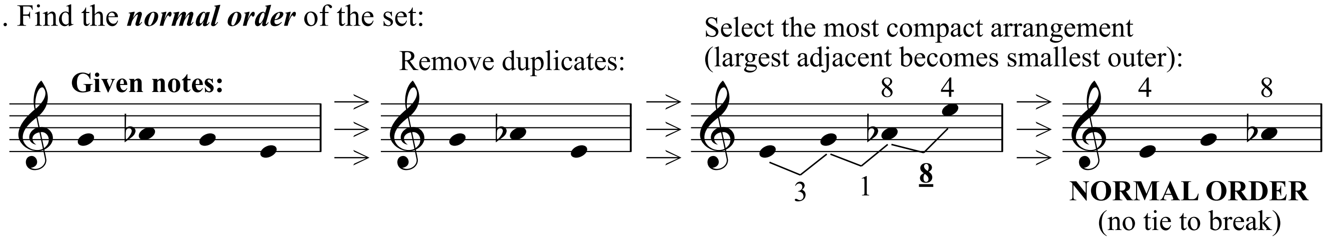
Best Normal Order

The ***best normal order*** of a set is one of two normal orders:

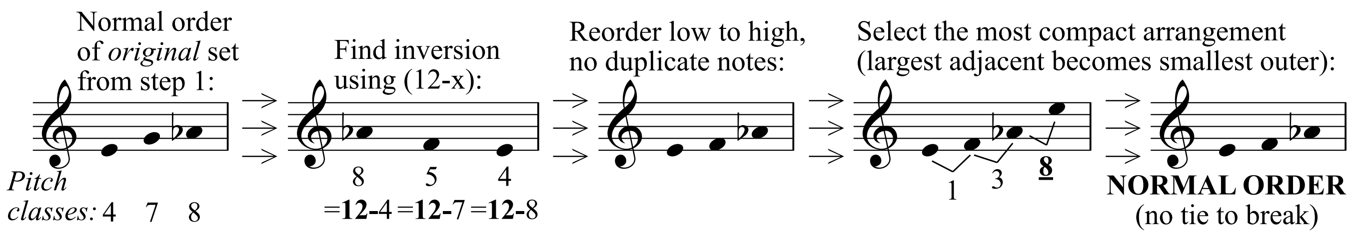
1. The normal order of the *given* set, **OR**
2. The normal order of the *inversion* of the given set (12-x for each pitch class, then do normal order).

Since these two normal orders will generally have the same outer intervals, choose the normal order with the smallest interval from the ***first*** note to the ***second-to-last*** note. If there is still a tie, check the ***third-to-last*** interval, and so on. Here is an example of the complete process:

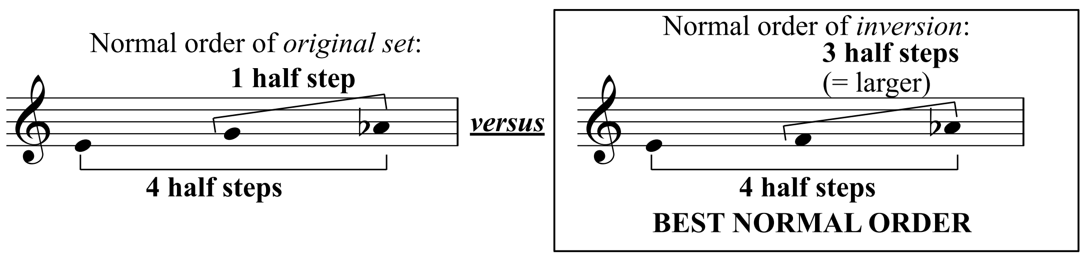
1. Find the ***normal order*** of the set:



2. Find the normal order of the ***inversion***:

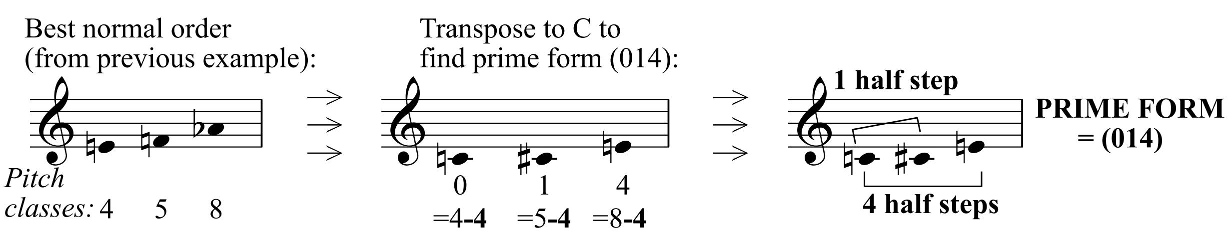


3. Choose the "best" of the two normal orders by comparing intervals. Since the outside interval (bottom to top) is the same, choose the **largest** interval from the ***second-to-last*** up to the ***last*** note:



Prime Form

Prime form = the best normal order as pitch class numbers transposed to start on C. *Since C=0, prime form reveals the **number of half steps** each pitch class is **above** the first one.* Prime form provides an easy shorthand for comparing the interval structure of sets throughout a piece of music.



In this case, the prime form is (014); the interval structure of the best normal order is thus 1 half step above the first pitch PLUS 4 half steps above the first pitch: (014).

6.10 Set Theory Simplified

Normal Order by the Clock (OPTIONAL)

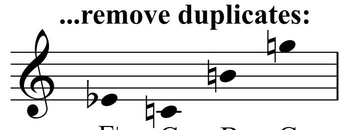
Some theorists prefer to find normal order using the pitch class numbers 0-11 arranged in a clockface:

1. **Remove duplicated notes** to get a list of unique pitches. ***Convert to pitch classes.***

Given these notes from some music...

→ → →

...remove duplicates:



E♭ C B G

Pitch classes: 3 0 11 7

2. Mark the pitches on the pitch-class "clockface," then start with the pitch class AFTER the largest gap. Below, the largest gap is from 3 to 7 or 7 to 11, so the options are [7, 11, 0, 3] <u>OR</u> [11, 0, 3, 7].

<u>0</u>

<u>11</u> 1

10 2

9 ***<u>3</u>***

8 4

<u>7</u> 5

6

3. To break a tie, select the version with the largest interval from the ***second-to-last*** pitch class to the ***last***. If there is still a tie, check the interval from the ***third-to-last*** to the ***last*** pitch class, and so on.

For [7, 11, 0, 3], the second-to-last to the last is 3-0 = 3 half steps.
For [11,0,3,7], the second-to-last to the last is 7-3 = 4 half steps, so **[11,0,3,7]** is the ***normal order.***

Unbreakable Ties for Normal Order and Best Normal Order

If the tie for ***normal order*** <u>cannot</u> be broken after comparing all the intervals, the set is ***transpositionally symmetric***. If the tie for ***BEST normal order*** <u>cannot</u> be broken after comparing all the intervals, the set is ***inversionally symmetric***. See *6.11 Symmetric Sets* for more details.

For any unbreakable tie, use the ***smallest starting pitch class number*** (that is, the one closest above C).

Normal Order by the Numbers (OPTIONAL)

It is possible to calculate the normal order entirely with pitch class numbers. Simply write down the pitch class numbers without the music notation and continue all calculations from there. If you look back at the preceding examples, you will notice that in each case the music notation is not necessary to do the calculations.

Format for Writing Sets as Text

Pitch class set (unordered): Use parentheses and commas, e.g. (E,G,A♭)
Normal order (or "normal form") OR best normal order: Use brackets & commas, e.g. [E,F,A♭]
Prime form: Parentheses, NO commas, e.g. (014). Use "T" for ten and "E" for eleven.